Alternative 2nd order modulators

- Large number of structures for $2^{nd}$ order noise-shaping with STF(z) = $1 / (z - a)$ or $z^2$
- Be careful to avoid delay-free loops
- Should have reasonable robustness against practical circuit requirements/limitations like finite opamp gain, gain, comparator delay (or quantizer delay), etc.

- Bose-Wooley Modulator

```
\[
\text{delaying integrator}
\]
```

- Settling requirements on the integrators (hence the opamps) is reduced.

\[
\text{NTF}(z) = \frac{(1-z)^2}{D(z)}
\]
\[
\text{STF}(z) = \frac{a_1 a_2 z^4}{D(z)}
\]

where,
\[
D(z) = (1-z)^2 + a_2 b z^{-1} (1-z) + a_1 a_2 z^{-2}
\]

for STF(z) = $z^2$ and NTF(z) = $(1-z)^2$, we have the conditions $a_1 a_2 = 1$ and $a_2 b = 2$.

\[
\Rightarrow D(z) = 1
\]

infinite solutions possible

\[
\text{Sol}^{a_1} : a_1 = a_2 = 1 \quad b = 2
\]
\[
\text{Sol}^{a_2} : a_1 = \frac{1}{2}, \quad a_2 = 2, \quad b = 1
\]

In actual design "Range-Scaling" eliminates these ambiguities in designs

Here, $U(z) - V(z) = \frac{(1-z^2) U(z)}{1-z^2} - \frac{(1-z)}{1-z^2} E(z)$

\[
\Rightarrow \text{check this spectrum}
\]
The Silva-Stensgaard Structure:

- Note the direct feed-forward path.
- \( V(z) = U(z) + (1-z^{-1}) E(z) \) \( \rightarrow \) Note \( U(z) \) is not delayed even when using delaying integrators.
- Input signal to the loop filter?
  \[
  = U(z) - V(z)
  = - (1-z^{-1})^2 E(z)
  \]
  contains only noise, no signal content.

Advantages:
1. Requirements on loop filter linearity are reduced. Lower power.
2. Signal swing at \( X \) is reduced. Lower SR requirements from the opamps.

- Output of the second-integrator:
  \(-z^2 E(z)\) can be directly used in a \( \text{HA-st} \) without any differencing.

Disadvantages:
1. Extra ADDER before the quantizer.
2. Passive adders using capacitors (for S/C design).
\[ \text{NTF}(z) = \frac{(1-z^{-1})^2}{A(z)} \quad \text{STF}(z) = \frac{B(z)}{A(z)} \]

\[ B(z) = b_1 + b_2(1-z^{-1}) + b_3(1-z^{-1})^2 \]

\[ A(z) = 1 + (a_1 + a_2 + a_3 - z^{-1}) z^{-1} + (1-a_1 - a_3) z^{-2} + a_3 z^{-3} \]

By using multiple feed-in and feedback paths, more flexibility is obtained for enhancing stability and dynamic range.

Which topology/architecture is the best for 2nd order modulator?

\( \implies \) What is the optimal NTF?

\( \implies \) Find an NTF which minimizes the IN BAND NOISE (IBN)

\[ |\text{NTF}(z)| = \frac{|(1-z^{-1})^2|}{|A(z)|} \approx \frac{\omega^2}{|A(z)|} \text{ for } \omega \ll \Omega \]

\[ = \frac{\omega^2}{|A(z)|} \]

\[ k = \frac{1}{|A(z)|} \]

\[ \text{Move the NTF zeros for } z = 1 \text{ to } z = e^{\pm j\alpha} \]

\[ \implies |\text{NTF}(z)| \text{ in the signal band } = k(\omega^2 - \omega^2) \]

\( \text{Used only in CT- DSM for Excess-Load Delay Compensation (Latv).
}
\[
\text{IBN} = \frac{\Delta^2}{12\pi} \int_0^\omega_B k(\omega^2 - \omega^2) \, d\omega
\]

\[
= \frac{\Delta^2 R^2}{12\pi} \int_0^\omega_B (\omega^2 - \omega^2) \, d\omega = \frac{\Delta^2 R^2}{12\pi} \cdot I(\omega)
\]

where \( I(\omega) = \int_0^\omega_B (\omega^2 - \omega^2) \, d\omega \)

\[
\text{for the least IBN, } I(\omega) \text{ must be minimized}
\]

\[
\Rightarrow \frac{dI(\omega)}{d\omega} = 0
\]

\[
\Rightarrow \int_0^\omega_B \frac{d}{d\omega}(\omega^2 - \omega^2) \, d\omega = 0
\]

\[
\Rightarrow \omega_B \int_0^\omega_B (\omega^2 - \omega^2) \, d\omega = 0
\]

\[
\Rightarrow 4\omega_B \int_0^\omega_B (\omega^2 - \omega^2) \, d\omega = 0
\]

\[
\Rightarrow \frac{\omega_B^2}{3} - \omega_B^2 = 0
\]

\[
\Rightarrow \omega_B = \frac{\omega_B}{\sqrt{3}}
\]

Now find, \( \frac{I(0)}{I(\omega_B)} = \frac{9}{4} \Rightarrow \text{SNIR improvement} = \log_{10}(9/4) = 3.5 \text{ dB} \)

Now, what if we also optimize the pole locations?

L) MATLAB based design using exhaustive search.

Optimal denominator

\[
A_{opt}(z) = 1 - 0.5z^{-1} + 0.1z^{-2}
\]

\( \text{LDA 6 dB higher SNIR.} \)

*We will study this algorithm in detail later.*
Describing Function Analysis

Generalized Block Diagram

\[ Y(z) = L_0(z)U(z) + L_1(z)V(z) \]

where,

\[ L_0(z) = \frac{STF(z)}{NTF(z)} \]
\[ L_1(z) = \frac{NTF(z) - 1}{NTF(z)} \]

For the whole modulator:

\[ V(z) = Y(z) + E(z) \]
\[ = L_0(z)U(z) + L_1(z)V(z) + E(z) \]

\[ V(z) = \frac{L_0(z)}{1 - L_1(z)} U(z) + \frac{L_1(z)}{1 - L_1(z)} E(z) \]

So far we have assumed linear Model of noise.

Loop-filter

\[ L_0 \text{ 2 input, 1 output} \]

\[ U \rightarrow L_0(z) \rightarrow y \]

\[ V \rightarrow L_1(z) \rightarrow y \]

* For Second-order DSM

\[ L_0(z) = \frac{1}{(z-1)^2} \]
\[ L_1(z) = -z^{-1} \frac{1}{1-z^{-1}} \]

* For special cases:

\[ L_0(z) = L_1(z) = L(z) \]

Ex., first-order DSM

So far, a linear model. But how to model the quantizer so as to understand the effects of its non-linearity.

- Overload (or saturation) of the quantizer causes instability:
  - When input exceeds the range of the quantizer, the output of the quantizer doesn’t change at all.
  - Feedback breaks down!

- Definition of stability:
  - for LTI systems, bounded input \( \Rightarrow \) bounded output (BIBO)
  \[ \sum_{n} |h(n)| < \infty \]
ECE 697 Delta-Sigma Converters Design

Lecture#9 Slides

Vishal Saxena
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$NTF(z) = (1 - z^{-1})^2$

File: Second_Order_DSM_Zero_Opt.m
Set variable opt=0.
2nd order DSM: contd.

SNDR = 73.8 dB
ENOB = 11.97 bits @OSR = 32
2\textsuperscript{nd} order DSM: NTF Zero Optimization

\[ NTF(\mathbf{z}) = (1 - e^{j0.06 \mathbf{z}^{-1}})(1 - e^{-j0.06 \mathbf{z}^{-1}}) \]

File: Second_Order_DSM_Zero_Opt.m
Set variable opt=1.

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• 5.5 dB increase in SQNR.
N Torrent Pole (if any) optimization to be discussed later.

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