2nd order DSM

\[ V(z) = u(z) + \left(1 - z^{-1}\right) e(z) \]

V(z) = \frac{y}{1-z^{-1}}

"Double-differential hysteresis of quantization noise"

\[ \text{NTF}(z) = \left(1 - z^{-1}\right) \]

Band quantization noise

\[ \text{IBN} = \frac{\Delta^2}{12\pi} \left( \frac{\pi}{\text{OSR}} \right)^3 \int_0^{\pi/\text{OSR}} |1 - e^{-j\omega}|^2 d\omega = \frac{\Delta^2}{12\pi} \left( \frac{\pi}{\text{OSR}} \right)^3 \int_0^{\pi/\text{OSR}} \omega^4 d\omega \]

\[ = \frac{\Delta^2}{12\pi} \cdot \frac{\pi^5}{5} \cdot \left( \frac{\pi}{\text{OSR}} \right)^3 \]

\[ = \frac{\Delta^2 \pi^4 \text{OSR}^5}{60} \]

\[ 2 \times \text{OSR} \approx 15 \text{dB} \rightarrow \text{SNR} \approx 2.5 \text{ bit} \rightarrow \text{in 8-bit resolution} \]
Example: quantize \( N_0 \) 4 bits resolution

\[ ASR = 6.4 \]

\[ N_{\text{inc}} = 2.5 \log_2(64) = 6 \cdot \frac{5}{2} = 15 \text{ bits} \]

\[ N_{\text{eq}} = N_0 + N_{\text{inc}} = 4 + 15 = 19 \text{ bits} \]

\[ \Rightarrow 16 \text{ levels } \Delta E^2 \rightarrow 2^{19} = 512 \times 10^3 \text{ levels} \]

Can't get this much resolution with Nyquist rate ADC's.

\[ N\!F\!T(z) = (1-z^{-1})^2 \]

\[ h[n] = [1, -2, 1]. \]

\[ N\!F\!T \text{ gain at } \omega = \pi \]

\[ = |N\!F\!T(e^{j\omega})| \omega = \pi \]

\[ = \sum_n (\pi)^n h(n) = 4. \]

\[ \sum_{n=0}^{\infty} |h[n]| \approx \rightarrow \text{ we'll see later} \]

\[ \text{lots of gain at the frequencies where we wish the quantization noise to be low very low.} \]
move the delay into the loop filter

⇒ No delayfree loops!

⇒ first sample of the impulse response = 1

⇒ no zero-delay loops.

Any output comes out with at least a unit delay ⇒ first sample is always \( s[n] \).

Ex. \[ \text{NTF}(z) = 1 - z^{-1} \]

\[ \text{Ex.} \quad \text{NTF}(z) = (1 - z^{-1})^2 = 1 - 2z^{-1} + z^{-2} \]

Notice that:

\[ \sum_{n} h[n] = 0 \Rightarrow \text{dc gain} = 0 \Rightarrow \text{High-pass response} \]

⇒ NTF response is always a HP response

⇒ RDJ = 1

and \( \sum_{n} h[n] = 0. \)
\[ H(z) = 1 \Rightarrow NTF(z) = \frac{1}{H(z)} \]

\[ NTF(z) = H[z] + H[z]z^{-1} + H[z]z^{-2} + \ldots \]

\[ \Rightarrow NTF(z^{-\infty}) = H[0] = 1. \]

If \( H[0] \neq 1 \Rightarrow \) NOT a physically realizable NTF.

Further understanding of the quantization noise in the loop.

Where \( Y(z) = V(z) - E(z) = STF(z) \cdot U(z) + NTF(z) \cdot E(z) - E(z) \)

\[ \Rightarrow STF(z) \cdot U(z) + (NTF(z) - 1) \cdot E(z) \]

In the time-domain:

- \( STF \equiv 1 \) at low frequencies
  \Rightarrow input will appear without any change at the output of the loop filter.

- How about the quantization noise?

\[ NTF(z) \downarrow \Rightarrow H[n] - S[n]. \]

The quantization noise is injected as \( E[n] \) into the loop and appears back at the loop filter output \( y[n] \).
Ex.

\[ \text{NTF}(z) = 1 - z^{-1} \]

IBG = \omega

OBC = 2

\[ \text{NTF}(z) = (1 - z^{-1})^2 \]

IBG = \omega^2

OBC = 4

SEE MATLAB

"wiggling is larger."

"AccumDataDem.m"

\[ \text{firstOrderDem.m} \]

\[ z^2 \text{ order } (1 - z^{-1})^2 \]

1 LSB jumps

Larger 2^2 LSB jumps

More OBC \Rightarrow \text{larger jumps in terms of the LSBs.}

- How to find the jum magnitude \( \delta \) from the NTF(2)?

L\( \delta \) gain by the maximum accumulation of the quantization error at the output of the loop filter.

\[ y[n] = u[n] + e[n] \otimes (h[n] - 1) \]

Consider only noise

\[ \Rightarrow \text{noise} = e[n] \otimes (h[n] - 1) = e[n] \otimes g[n] \]

where \( g[n] = h[n] - 1 \).
Accumulated noise \( = e[n] \circ (-h[n-1]) = e[n] \circ g[n] \)
\[
= \sum_{i=0}^{\phi} g[i] e[n-i] \quad g[n] \text{ is causal}
\]
\[
\leq \sum_{i=0}^{\phi} |g[i]| |e[n-i]|.
\]
\[
\leq \frac{\Delta}{2} \sum_{i=0}^{\phi} |g[i]| \quad \rightarrow \quad ||g[n]||_1 \quad 1\text{-norm of } g[n].
\]
\[
= \frac{\Delta}{2} \sum_{i=0}^{\phi} |g[i]| \quad \rightarrow \quad \frac{\Delta}{2} \cdot ||h[n]-1||_1 \quad \text{Key quantity.}
\]

After some "hand-waving" intuition

Max. LSB jump = 2 \times \text{ Accumulated noise} = \Delta \cdot ||h[n]-1||_1

Example: ① \( NTF(z) = (1-z^{-1}) \)

Max. LSB jump = \( \Delta \cdot 1 = \Delta \cdot 1 = 1 \text{ LSB} \)

② \( NTF(z) = (1-z^{-1})^2 = 1-2z^{-1}+z^{-2} \)

Max. LSB jump = \( \Delta \cdot (121+111) = 3\Delta = 3 \text{ LSB's} \)

How about third- or higher order?

\( NTF(z) = (1-z^{-1})^N \)

\( \Rightarrow \text{IBN} = \frac{\Delta^2}{12\pi} \int_0^{\pi/\text{OSR}} \omega^2 \text{d}\omega \)

\[
= \frac{\Delta^2 \cdot \pi}{12\pi} \left[ \frac{2^{N+1}}{(2N+1)} \right]_{0}^{\pi/\text{OSR}}
\]

\[
= \frac{\Delta^2 \pi^{2N+1}}{12 \pi (2N+1)} \cdot \text{OSR}
\]

\( \Rightarrow 3 \times (2N+1) \text{ dB} \) per 2 \times \text{OSR}

\( \Rightarrow (N+\frac{1}{2}) \text{ bit increase in resolution} \)

for 2 \times \text{OSR}.

Stability issues! will come back to it later.
ECE 697 Delta-Sigma Converters Design

Lecture#8 Slides

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Delta-Sigma (ΔΣ or DS) Modulation

- Use oversampling ($f_s = 2 \cdot OSR \cdot BW$) to shape the quantization noise out of the signal band.
- Use low-resolution ADC and DAC to higher much higher resolution
  - In MATLAB, Quantizer = ADC + DAC
- Digitally filter away the out-of-band shaped (modulated) noise.
- Trades-off SNR with oversampling ratio.

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First-order Noise Shaping

DSM time-domain Simulation

DSM Output Spectrum

File: First_Order_DSM.m

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Second-order Noise Shaping

File: Second_Order_DSM.m

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Comparison: 1\textsuperscript{st} and 2\textsuperscript{nd} order modulator waveforms

- NTF(z) = (1- z^{-1})
- OBG = 2
- Max LSB jump = 1

- NTF(z) = (1- z^{-1})^2
- OBG = 4
- Max LSB jump = 3
Third-order Noise Shaping (trivial design)

- \( \text{NTF}(z) = (1-z^{-1})^3 \)
- \( \text{OBG} = 8 \), Full-scale input.
- Unstable after few samples (look at \( y[n] \) blowing up).
  - Worst for a single-bit quantizer.

File: Third_Order_DSM.m
Third-order Noise Shaping contd.

- Input amplitude = 0.5 \cdot FS
- Signal dependent stability.
  - Need to develop intuition for modulator stability.
  - Reference: Stability theory from the Yellow book of delta-sigma.

File: Third_Order_DSM.m