Signal estimation using DSP/Matlab.

Example: characterizing the distortion of a S/H using a single tone input.

\[ \text{Vin} \xrightarrow{\text{Non-linear System}} \text{Vn} \xrightarrow{\text{Ideal S/H}} \text{Vn} \]

Use a discrete time periodic sequence with period \( N \).

\[ V[n] = V[n+N] \]

Then \( V[n] \) can be represented as DFS.

\[ V[n] = \sum_{k=-\infty}^{\infty} V[k] e^{j2\pi kn/N} \]

\( V[k] \) are easily computed using FFT.

- \( N \rightarrow \text{record length} \)
- \( N \rightarrow \text{FFT size} \)
- \( N \rightarrow \text{time in continuous time axis} \)
- \( \frac{N}{f_s} \rightarrow \text{resolution of the FFT} \)
- \( \frac{f_s}{N} \rightarrow \text{spacing between the tones} \)

P.S.: M \(
\begin{array}{c}
\text{record length} = \text{size of the data collected from simulation or measurement} \\
\text{FFT size} = M, \text{ i.e. } N = M
\end{array}
\)
\[ V_{in}(t) \rightarrow V_{x}(t) \rightarrow \frac{nT_s}{f_s} \rightarrow V_{out}[n] \]

\[ V_m = A \sin(2\pi f_m t) = A_m \sum_k A_k e^{j2\pi kf_m \frac{t}{T_s}} \]

\[ V_n = \sum_k A_k e^{j2\pi kf_m \frac{t}{T_s}} \quad a_1 \rightarrow \text{fundamental} \quad a_{2,3,\ldots} \rightarrow \text{harmonics} \]

\[ V_{out}[n] = V_{out}(\frac{n}{T_s}) = \sum_k A_k e^{j2\pi k f_m \frac{n}{T_s}} \]

\[ V_{out}[n] \text{ is periodic only when} \]

\[ e^{j2\pi k f_m \frac{n+N}{T_s}} = e^{j2\pi k f_m \frac{n}{T_s}} \]

\[ 2\pi f_m \frac{n}{T_s} = 2m\pi, \quad m \in \mathbb{Z} \]

\[ \frac{f_m}{f_s} = \frac{m}{2N} \]

only then \( V_{out}[n] \) is a periodic sequence and DFS is valid.

In time-domain:

\[ \frac{m}{f_m} = \frac{N}{f_s} \rightarrow 'm' \text{ cycles of } f_m = 'N' \text{ cycles of } f_s \]

\[ \text{If } (i) \text{ is satisfied } V_{out}[n] \text{ can be expressed as discrete Fourier Series.} \]

\[ \frac{T_s}{N} = \text{FFT resolution} \]

\[ \frac{f_s}{N} = \text{'bin' resolution} \]

Each tone is called a "bin".

\[ 'N' \text{ bins for an N-point FFT.} \]
for $f_{in} = \frac{m}{N} f_s$

with distortion → component at multiple of $\frac{m f_s}{N}$

choose $f_{in}$ and $f_s$ carefully else the sampled sequence will not be periodic and the FFT will not show the correct results.

Example:

select $f_{in} = \frac{m f_s}{N} = \frac{5 f_s}{4}$,

harmonic components → $0, \frac{5 f_s}{4}, 2\frac{5 f_s}{4}, 3\frac{5 f_s}{4}, \ldots$

record length = $N = 4$.

For $\frac{N}{f_s} = \frac{2 f_s}{8}$, is record length equal to 87?

→ make sure that $m$ & $N(=N)$ are relatively prime.

Distortion: fold-back.
BFT (or FFT).

Conjugate symmetry

\[ x^*[k] = x[N-k] \]

\[ |x[k]| = |x[N-k]| \]

and \[ /x[k] = -/x[N-k] \]

For \( f_{m} = \pm 1/4 \) not distinguishable

\[ \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{5}{4} \]

Will not yield correct distortion analysis.

\( f_{m} \) not just enough that if

\[ f_{m} = \frac{m}{N} \]

necessary but not sufficient

\[ f_{m} = \frac{m}{N} \] is prime w.r.t \( N \) (and \( N \) is large compared to \( m \) so that the harmonics don't alias back to the fundamental

Also choose \( N = 2^{b} \) for FFT computation

Eq. \( f_{m} = \frac{m}{2^{b}} f_{s} = \frac{m}{1024} f_{s} \).

See "FFT Demod. m" Here \( f_{m} = \frac{129}{1024} f_{s} \).
"FFT Demo1.m"

- Absolute value of the FFT doesn't matter due to the normalizing factors.

fundamental tone

Noise floor due to the limited precision of the computer.
What happens when $f_{\text{fin}} = \frac{129.01}{1024} f_s$?

- Bins which are supposed to be zero are now filled up, and are only 80/90 dB lower.
- Signal is not periodic
- $\frac{m}{f_{\text{fin}}} \neq \frac{N}{f_s}$

(FFT Recap)

Consider a finite length sequence $x[n]$ with length $N$.

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n] e^{-j\omega n} \rightarrow \text{DTFT}$$

$$\hat{x}[n] = \sum_{n=-\infty}^{\infty} x[n-N]$$

$$X[k] \xrightarrow{\text{DFS}} \hat{x}[k] \leftarrow \text{periodic}$$

$$x[n] \xrightarrow{\text{DFT}} X[k] \leftarrow \text{fixed length } N$$

DFT is sampled DTFT with $\omega = \frac{2\pi k}{N}$.

But $x[n] = \sin(2\pi f_{\text{fin}} n)$ is a periodic signal of infinite length

and we restrict it to a length $N$.

The signal $x[n]$ is windowed.
\[ p[n] = x[n] \cdot w[n] \quad \xrightarrow{\text{DTFT}} \quad X(e^{j\omega}) \otimes W(e^{j\omega}) \]

\[ \Rightarrow e^{-j\omega \left( \frac{N-1}{2} \right)} \cdot \frac{\sin \left( \frac{\omega N}{2} \right)}{\sin \left( \frac{\omega}{2} \right)} \]

\[ P(e^{j\omega}) = X(e^{j\omega}) \otimes W(e^{j\omega}) \]

\[ \Rightarrow \text{If } f_{in} = \frac{m}{N} f_s, \text{ i.e. the input is rationally related to the clock frequency, we get a single peak and other bins are zero.} \]

\[ \Rightarrow \text{DFT computes DFT properly.} \]
What if \( \frac{\sin \omega_0}{\omega_0} \neq \frac{m}{M} \)?

The closer \( \frac{\sin \omega_0}{\omega_0} \rightarrow \frac{m}{M} \), the Eiffel tower becomes taller.

[see Matlab code FFTdemo2].

- Synchronous sampling
- Coherent sampling

Time domain understanding

![Diagram of coherent and non-coherent signals with discontinuity in signal]
Temporal discontinuity while taking DFS causes FFT leakage.

Discontinuity → step function → has all frequency components.

How to subdue the effect of the discontinuity in non-coherent sampling?

1. Signal reconstruction:

   Adjust 'N' such that 'a' has full cycles of $x[n]$
   - Impractical to implement.

2. Alternating the discontinuity.

   Use custom windows instead of rectangular windows.

   - Give lesser emphasis on the ends and more importance to the segment in the middle.
   - A large number of windows are reported in literature and are available in Matlab.
Rectangular window

\[ W[n] \]

\[ \frac{\sin \left( \frac{N\pi n}{N} \right)}{\sin \left( \frac{\pi n}{N} \right)} \]

Triangular window

\[ W[n] \]

\[ \frac{\sin^2 \left( \frac{N\pi n}{4N} \right)}{\sin^2 \left( \frac{\pi n}{4} \right)} \]

Bartlett window

(Or make it smoother)

Double the main-lobe width
Longer side-lobe suppression

Number of signal bins, \( N_b = 3 \)

Raised cosine window (Hann, Hanning)

\[ W[n] = \frac{1}{2} \left[ 1 - \cos \left( \frac{2\pi n}{N} \right) \right], \quad 0 \leq n \leq N - 1 \]

\[ x[n] = A \sin \left( \frac{2\pi}{1024} \cdot 129n \right) \]

\[ \sin A \approx \sin B = \frac{1}{2} \left[ \sin (A+B) + \sin (A-B) \right] \]

\[ x[n] = A \sin \left( \frac{2\pi}{1024} \cdot 129n \right) - \frac{A}{4} \sin \left( \frac{2\pi}{1024} \cdot 128n \right) - \frac{A}{4} \sin \left( \frac{2\pi}{1024} \cdot 130n \right) \]

Signal bins = \( 129, 129, 130 \)

\[ N_b = 3 \]
Using Hanning window,

if the sampling is not coherent,

the energy of the tone disperses into the side bins.

But with Hanning window, the FFT leakage is a lot smaller than with the rectangular window.

Check with Matlab.

\[
\text{for } f_m = \frac{129.01}{1024} \text{ fs} \\
\text{Rectangular window} \\
\text{Hanning window}
\]

\( k \)

Also, the harmonics don't leak and smear into each other as much as with rect windows.

Other windows:

**Blackmann-Harris Window**: 

\[
W[n] = a_0 + a_1 \cos\left(\frac{2\pi n}{N}\right) + a_2 \cos\left(\frac{2\pi n}{2N}\right) + a_3 \cos\left(\frac{4\pi n}{3N}\right)
\]

\(-\frac{N}{2} \leq n \leq \frac{N}{2}\)

length \( L = N + 1 \)

\[
a_0 = 0.25876 \\
a_1 = 0.48829 \\
a_2 = 0.14128 \\
a_3 = 0.01168
\]

Look up in Matlab

\[
W[n] \\
\text{B-H window} \\
\text{Hann}
\]

\( n \)

\( \Rightarrow \) Maximum Sidelobe Suppression (but main lobe width is larger)

Matlab:

\[ L = 32; \]

\[ \text{WN} = \text{blackmannharris}(L). \]
ECE 697 Delta-Sigma Converters Design

Lecture#4 Slides

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Spectral Estimation
Coherent Sampling

\[
x[n] = \sin(2\pi \frac{f_{in}}{f_s} n)
\]

\[
\frac{f_{in}}{f_s} = \frac{129}{1024}
\]

file:FFTdemo1.m
Non-Coherent Sampling : FFT leakage

\[ \frac{f_{in}}{f_s} = \frac{129}{1024}, \frac{129.01}{1024}, \frac{129.001}{1024} \]

file:FFTdemo2.m

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FFT leakage contd.

\[ \frac{f_{in}}{f_s} = \frac{129}{1024}, \frac{129.01}{1024}, \frac{129.001}{1024} \]

file:FFTdemo2.m
% Compare Rect, Bartlett and Hann windows

L = 32;

wvtool(rectwin(L), bartlett(L), ds_hann(L));

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Spectral Windows contd.

% Compare Blackman-Harris and Hann windows
L = 32;
wvtool(blackmanharris(L), ds_hann(L));

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FFT with Windowing

\[ \frac{f_{in}}{f_s} = \frac{129.01}{1024} \]

file:FFTdemo_windowing.m

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References